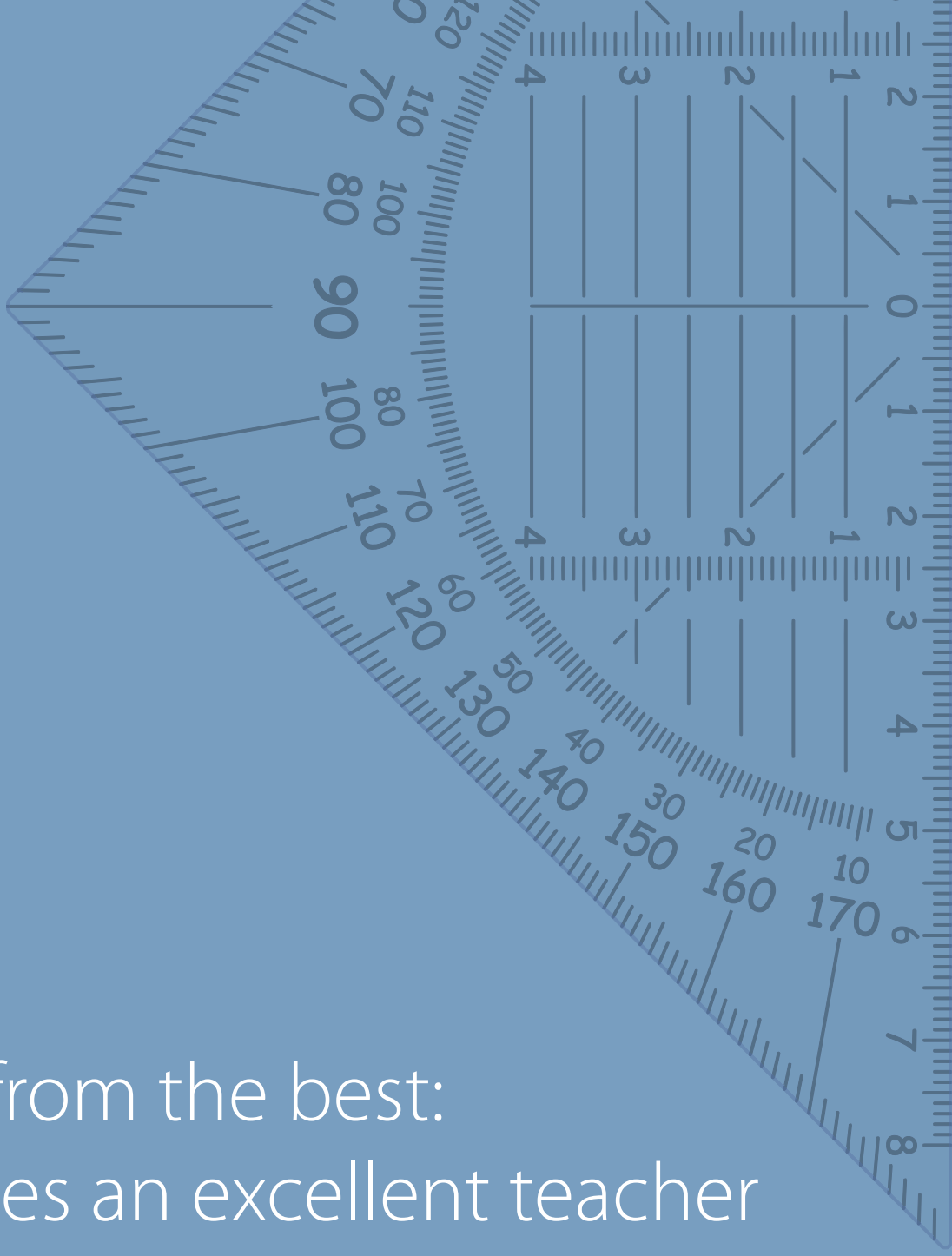

Learning from the best: what makes an excellent teacher of mathematics?

By Dr Bronwyn Hinz, Dr Lyndon Walker, and Mr Michael Witter

Pivot
Transforming teaching practice

About Pivot

Pivot Professional Learning is an organization that aims to transform teaching practice through a suite of data-rich tools including our flagship Student Perception Survey. Pivot's Student Perception Survey asks students to provide feedback on teaching practice at the individual classroom level, which informs teaching at the individual, department, school, network and system level. Results are confidential, available the day after surveys close, and broken down in a meaningful way for each level of user. This makes it easy to identify and respond to the changing needs of each class, year level, department or school.

Pivot's student survey data helps teachers, school leaders and executives to pinpoint strengths for sharing, and growth priorities for development.

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Learning from the best: what makes an excellent teacher of mathematics?[1]

By Dr Bronwyn Hinz, Dr Lyndon Walker, and Mr Michael Witter[2]

Executive summary

Australia has many excellent mathematics teachers. However, it also faces many challenges on a national scale: declining mathematics results; limited engagement among Australian students; a severe shortage of qualified mathematics teachers; low levels of numeracy and a lack of confidence in mathematics across the general population (O'Connor and Thomas 2019; IAEEA 2015; FYA and AlphaBeta 2017; Thomson et al. 2017).

In order to enhance mathematics instruction, engagement and outcomes across Australia, it is imperative we learn from our top mathematics teachers, as well as invest in high-quality pre-service teacher education and the provision of ongoing and relevant professional learning throughout a teacher's career. But what are the key indicators and practices Australia should seek to spread?

Pivot Professional Learning (Pivot), in consultation with the Australian Association of Mathematics Teachers (AAMT), undertook an exploratory study of the characteristics of Australian mathematics teachers to better understand what helps and hinders great teaching so that we can learn from the best.

Method

Using a short, opt-in questionnaire, we collected information on factors the literacture suggest could make the biggest contribution to quality teaching - qualifications, experience, professional learning, teaching beliefs, self-efficacy and collective efficacy. 986 teacher from around Australia particpated in this questionnaire. Results data from the Pivot Student Survey[3] for each teacher, where available, was then accessed and linked to the questionnaire (n=673), with teachers' consent, to explore relationships between items. Those teachers with the highest aggregate scores from the Pivot survey we identified as 'Top Teachers'. Follow-up qualitative interviews using a set of semi-structured questions were then held with a small but representative sample of Top Teachers. These interviews were to gain greater insights into features of their training, professional learning, pedagogical beliefs and teaching practices that appear to make the biggest difference in supporting students to learn mathematics.

1 This paper was presented at the AAMT Conference, Brisbane, 9 July 2019, in session titled 'Great, confident maths teaching: what are the indicators, training and PL supports and practices?'

2 Dr Bronwyn Hinz is Director of Research and Development at Pivot Professional Learning, and Honorary Fellow at the Melbourne Graduate School of Education. Dr Lyndon Walker is a former academic, OLT Citation holder, and statistical consultant for Veritate Data Science. Michael Witter is Director of National Curriculum at Teach For Australia and a PhD candidate with Prof. John Hattie focusing on dimensions of teacher quality. The authors express their deep appreciation to all 986 study participants, and also to the AAMT and seven external reviewers (four of whom were expert math teachers) who each provided useful feedback that contributed to the accuracy, clarity and overall quality of this paper. The authors' take responsibility for any remaining faults.

3 Pivot's evidence-based, validated tool is an established method for measuring student perceptions of teacher effectiveness against each of the five Australian Professional Standards for teachers developed by the Australian Institute of Teaching and School Leadership (AITSL Standards). It consists of a five-minute, 25-item survey completed by school students on their perception of the teaching practices they experience in each classroom, with confidential, next-day reports available for each teacher, and aggregated results reports available for school leaders. (Available as Appendix A2)

Key Insights

Although this was an exploratory study, the large number of participants and statistically significant findings, substantiated with qualitative interviews, provide insights into the attributes and training of Top Teachers of mathematics. It is our hope to explore these more deeply in follow-up studies.

Top Teachers

- **Have strong connections with their students**, characterised by mutual respect and high expectations. They understand where each student is at, how to support them, engage them and stretch them. This student-teacher connection appears to be a higher-order factor for great mathematics teaching. This supports the idea that students filter their learning experiences and perceptions of quality teaching in part through the quality of student-teacher interactions (Hamre et al 2014; Kufheld 2017).
- **Emphasise the importance for mathematics lessons to include collaborative work** (where students can build and reinforce their understanding from the understanding of their peers), "hands-on" **problem-solving, and linking the learning to real life** in order to trigger and sustain student engagement. Top Teachers use their strong connections with students to design and adapt their lessons to effectively include these elements.
- **Recognise that deep learning (exploring concepts) and surface learning (recalling facts) are both important** and connected in building mathematical knowledge and skills.

Professional learning and teaching experience

- **Newer teachers had lower results for classroom management indicators**, which can make teaching and learning more difficult. While this is true, there were novice and experienced Top Teachers (as measured by Pivot's Student Perception Survey) which suggests that both more-experienced and less experienced teachers can learn from each other.
- **Teachers that had participated in maths-specific professional learning were more confident** than teachers that had not done so when it came to:
  - Demonstrating, modelling and explaining skills and concepts to students
  - Effectively using a variety of assessment and feedback strategies
  - Accessing the resources they need.
- **Mathematics teachers see professional learning as most valuable when they can identify its relevance for their students and classes they teach.**
- **Teachers reported the best types of professional learning were ongoing and promoted both action and reflection**, both as an individual teacher, and with colleagues, to support and fine-tune implementation of PL strategies in their classes.
- Teachers who were **members of professional associations** - such as the Australian Association of Mathematics Teachers - were more likely than non-members to agree that they **can access the resources** they need to teach effectively.

Qualifications and other characteristics

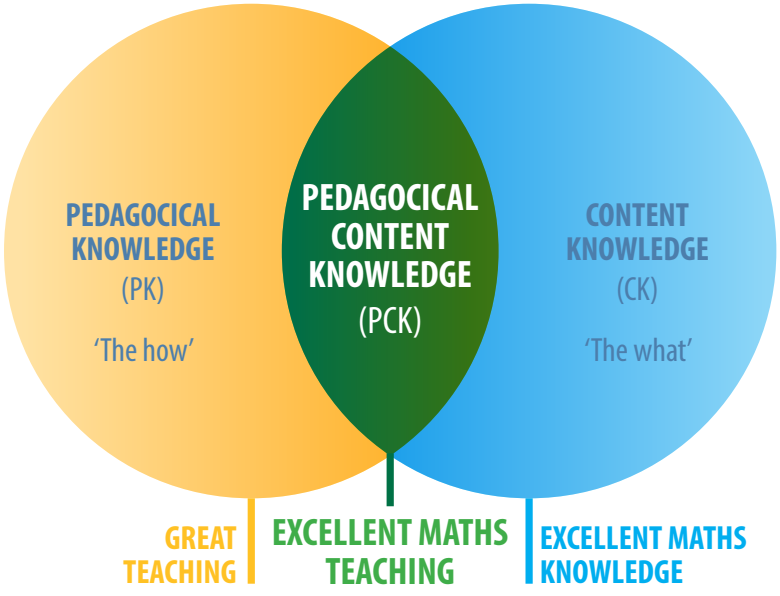
- **The uniting characteristic of Top Teachers was their ability to form strong connections with students**, which was the foundation for creating engaging lessons, determining the extent to which each student understood what was taught, and identifying what supports or extension are required to better enable each student to master the mathematical concepts and skills. This was a stronger indicator than their highest qualification, participation in professional learning, or years of teaching experience.
- **There were no significant differences between the average Pivot scores of male and female teachers**.
- **Teachers with higher degrees in mathematics reported greater confidence** in demonstrating, modelling and explaining mathematical concepts and skills, and were more likely to agree that their teaching helped students to apply information in new scenarios. However, this confidence did not always translate to the classroom experiences of their students. This is significant in the context of out-of-field teaching, and connects to the point below.
- **Mathematics expertise and teaching expertise are different but not mutually exclusive**. Excellent mathematics teachers need both sets of expertise in order to combine deep conceptual knowledge with the additional knowledge and skills to articulate these concepts in different ways, and to provide a myriad of opportunities for students to practise and apply mathematics in a way that triggers and sustained their interest. This also flows from strong student-teacher connections.

Implications

This study reinforced that Australia could do better at sharing and building excellent mathematics teaching within schools, between schools, and across systems. The study also reinforced that mathematics expertise (deep mathematical content knowledge) and teaching expertise (deep pedagogical knowledge) are different skills and that excellent teaching requires both. This has implications for both pre-service teacher education and ongoing professional development and learning.

Figure 1

Excellent mathematical teachers combine maths expertise with teaching expertise enabled through strong connections with their students



Recommendations

Professional associations and school systems can:

- Create and curate an online space and repository for mathematics teachers to share and discuss teaching resources, strategies, and professional learning from Top Teachers. While this could include contributions by relevant academics and other mathematics professionals, the content would be mostly by teachers, for teachers. This has the potential to increase teacher use and engagement, as they can more readily see relevance when it comes from fellow teachers.
- Develop professional learning opportunities and programs that support teachers to build strong, interactive connections with their students in their mathematics classes.
- Continue to promote and support professional learning that supports the cultivation of collaboration, creativity and critical thinking as they apply in mathematics.
- Recognise that quality teaching integrates surface knowledge (of facts) and deep learning (exploring concepts).

Schools can:

- Support early career teachers to build their classroom management skills, as this is a critical factor in being able to teach well. This could include more intensive scaffolding and mentoring of new teachers for the first five years, and more team-teaching (where teachers teach in pairs or small groups), which provides opportunities for more experienced teachers to share their expertise in classroom management and student engagement, and newer teachers to share research and strategies learnt in their pre-service education and teaching rounds.
- Encourage all mathematics teachers to participate in both formal and informal collegial observations (with clear focus, debrief and follow-up plan and actions).
- Support teachers to collaborate in their use and development of teaching resources and strategies, as well as to share relevant research they've read, as part of their ongoing professional learning.
- Identify and celebrate existing Top Teaching teams as established by those teachers' students (for example using Pivot survey data and other achievement data) to reveal the most effective practices for teaching different concepts, different year levels and students of differing ability levels.
- Recognise that highly experienced teachers and new teachers can both be excellent teachers of mathematics, and encourage all teachers and teams to share strategies for articulating key concepts and engaging students in maths.
- Connect their mathematics department with that of other schools in a mathematics Community of Practice, to share strategies, resources and to build collective efficacy.

Governments, school systems and partner bodies can:

- Work alongside schools to provide additional support, resourcing and development opportunities for early career teachers, including team teaching where this is desired by the school.
- Support out-of-field mathematics teachers to upskill, and other teachers wanting to consolidate or extend their knowledge, through supporting their access to specialised, bridging courses.
- Continue to promote and support - through policies, curriculums and resourcing - the importance of student voice, and of collaboration, creativity and critical thinking in mathematics and related subjects.
- Curriculum authorities could consider more emphasis on multidisciplinary approaches to learning mathematics, and guidance to support this, as a way of encouraging engaging and contextualised mathematics learning.
- Consider reducing the amount of content in senior curriculums, to allow for deeper exploration and mastery of key concepts, and continuation of best practice teaching and individualised approaches to support all students to achieve the best.

Introduction

Australia has many excellent mathematics teachers. But a variety of factors contribute to a situation where not every student or teacher can reach their full potential in learning or teaching this vital subject.

Pivot Professional Learning, in consultation with the Australian Association of Mathematics Teachers, decided to learn from the best in order to build and share the expertise of our top teachers, and build the mathematical skills and knowledge of our students. This paper presents the findings from our short, exploratory study seeking to identify characteristics of top teachers (qualifications, beliefs, practices, professional learning and more) and relationships between these and the learning experiences of their students. This was a mixed-methods study involving quantitative and qualitative components. Research instruments included a questionnaire of teachers, with items drawn from a review of the literature, results from a student perception survey on effective teaching practices, statistical analyses and qualitative interviews.

Results indicated that teachers that had participated in maths-specific professional learning, or held higher degrees in mathematics were more confident across a range of measures; that strong and positive student-teacher relationships are a foundation for highly effective teaching and learning, and that Top Teachers needed both deep conceptual knowledge and teaching expertise. These findings reinforce the importance of Pedagogical Content Knowledge, which enables teachers to provide interactive lessons with opportunities for collaboration, real-world problem solving and creativity, to build students' engagement, confidence, skills and knowledge in mathematics. This has implications for teachers, universities, schools, governments and other education stakeholders.

Background and literature review

The importance of effective teaching

Teaching effectiveness is the single most important in-school factor influencing student engagement and achievement (Hattie 2013). A large proportion of the variance that occurs in student achievement is attributable to their teachers, and teachers are probably the greatest school-related factor influencing student achievement gains (Sanders and Horn 1998, Darling-Hammond 2000, Rivkin, Hanushek and Kane 2005). The magnitude of the effect of the teacher on student achievement can be as much a full year's difference (Hanushek 1992). Rowe (2003) provided a strong evidence base demonstrating the importance of teacher effectiveness in influencing student outcomes in the Australian context, as did Leigh (2010) who found that teachers in the 75th percentile were achieving in three quarters of a year what a 25th percentile teacher achieved in a full year, while teachers in the 90th percentile achieved in a half-year what 10th percentile teachers achieved in a full year.

Teaching effectiveness and student achievement in Australia

Initiatives and investments in teacher effectiveness have featured strongly in state and Commonwealth education policies. Among other things, this has resulted in the creation of the Australian Institute of Teaching and School Leadership (AITSL) and its professional standards for teaching, as well as near-continuous reforms over the past decades to strengthen pre-service teacher education. Unfortunately, there is no clear evidence these reforms have improved academic outcomes for students in general, and specifically in mathematics. The 2015 PISA results indicated that Australian students' scores continue to decline from prior years, continuing a downward trend that has been occurring now for over a decade. Meanwhile data from TIMSS (Trends in International Mathematics and Science Study) show Australian students flatlining while other countries improve (Thomson, Wernert et al. 2016). Further to this, only slightly more than half of Australian students are achieving the National Proficient Standard in mathematical literacy (Thomson, De Bortoli et al. 2017).

Many factors contribute to these lackluster results. These factors include: poor policy coordination between state and Commonwealth levels of government; inadequate or poorly targeted funding (Hinz 2016); a severe shortage of qualified mathematics teachers (O'Connor and Thomas 2019); teacher and school leader workloads (OECD 2019); and poor or incomplete implementation at school and classroom levels, and miss-match between the program and the problem or context (Evidence for Learning 2019). An absence of evaluations or research exploring which of these factors most affected the policies seeking to improve teaching and student learning means we are left to speculate as to what reforms, programs and actions at all levels could make the biggest difference to teacher effectiveness and student learning growth in mathematics.

Measuring effective teaching

Teaching is a complex profession involving a number of interrelated skills and attributes and influenced by a number of contexts and other variables. We, the authors, take a complementary approach to measuring and understanding teacher effectiveness, reflecting this complexity and ongoing debate. In doing so, we follow other Australian and international researchers (Darling-Hammond 2000; Rice 2003, Wayne and Youngs 2003; Ingvarson and Rowe 2008; Naylor and Sayed 2014) in drawing upon Strong (2011), whose review of the research on this question identified four basic characterisations or conceptions of quality: teacher qualifications (including their degree, program and experience), personal attributes (such as kindliness, flexibility, patience), pedagogical skills and practices, and teacher effectiveness (the value added to student academic achievement).

Our exploratory study investigated which of these characteristics and indicators are most useful in understanding and supporting excellent mathematics teaching in Australia, with particular reference to experience (years teaching overall, and years teaching maths), highest relevant qualification, professional learning, attributes and beliefs (including self-efficacy and collective efficacy, and surface versus deep learning). We also explored the connections of these indicators with the learning experiences of their students as an indicator of quality teaching practices in these teachers' classrooms.

In doing so we drew upon the landmark MET study (2013) and more recent Wallace et al. study (2016) which found that carefully designed and statistically validated student perception surveys on specific teaching practices were found to be a reliable indicator of effective teaching practices and their impact on student performance. This finding has further reinforced by Australian research indicating that student feedback is among the "most powerful" kids of feedback for effective teaching (Hattie 2008); that successful systems of feedback including from students, can increase teacher effectiveness by as much as 30 per cent (Jensen 2011), and OECD research which found that student feedback surveys are a key element of teacher professional development systems in high performing countries such as Finland and Singapore (2013).

Method

Invitations to complete a short, online questionnaire on their mathematics teaching experience, training, and beliefs were sent by email to all teachers on Pivot Professional Learning's database who had taught a mathematics class in an Australian school in 2017, 2018 and 2019 (n=2683). These invitations were sent in March and April 2019. Information on the study with an invitation to participate was also distributed by the AAMT to their members.

This teacher questionnaire covered participants' teaching experience, qualifications, professional learning, teaching and learning beliefs, self-efficacy and collective efficacy *(Teacher questionnaire items are attached in Appendix A1)*. This questionnaire was designed by the authors, in consultation with the AAMT, to capture key items identified from the literature as having a potential bearing on teaching quality. A total of 986 mathematics teachers participated in this questionnaire, and came from all Australian states, territories and school sectors.

Results data from the Pivot student survey for each teacher, where available, were then accessed and linked to the questionnaire data (n=673), with teachers' consent, to explore relationships between items. *(Student survey items are attached as Appendix A2)*. While this study did not link this data to student performance data, we are confident the student survey instrument is a reliable indicator of effective teaching practies and of quality and effective teaching practices, due to extensive validity testing across Australia, sophisticated design of the instrument meeting all key criteria for such surveys, and their impact on student on student performance and engagement across a diversity of school contexts.

Descriptive statistics are presented in the form of means and percentages, with t-tests, chi-squared tests, analysis of variance (ANOVA), and multivariate multiple regression methods used to examine the bivariate and multivariate relationships between teacher demographics, teacher perceptions from the questionnaire, and student perceptions from the Pivot instrument. A combination of confirmatory and exploratory factor analysis was used to determine whether the various survey items could be grouped into broader categories.

Follow-up interviews were then conducted in June with a representative selection mathematics teachers with the highest aggregate Pivot student survey scores (i.e. the most effective as rated by their students based on teaching practices they experienced) to explore in greater depth the relationships found in the quantitative analysis. The 20 highest-scoring mathematics teachers who had also indicated interest in a follow-up interview were emailed invitations to be interviewed. Four teachers participated in semi-structured telephone interviews of 30-45 minutes. *(Interview questions are attached in Appendix A3.)* Of these, two were male, two were female, they had taught for 2 years, 20 years, 43 years, and 45 years, in government and non-government schools, and in metro and regional locations. This diversity reflects the pool of the 20 Top Teachers.

Participation in all elements of this study were voluntary, and participants were informed with a plain language statement, questionnaire preface, as well as in opening statement in research interviews that their participation was voluntary, that their data was confidential, and that they could withdraw at any time without prejudice.

Results and discussion

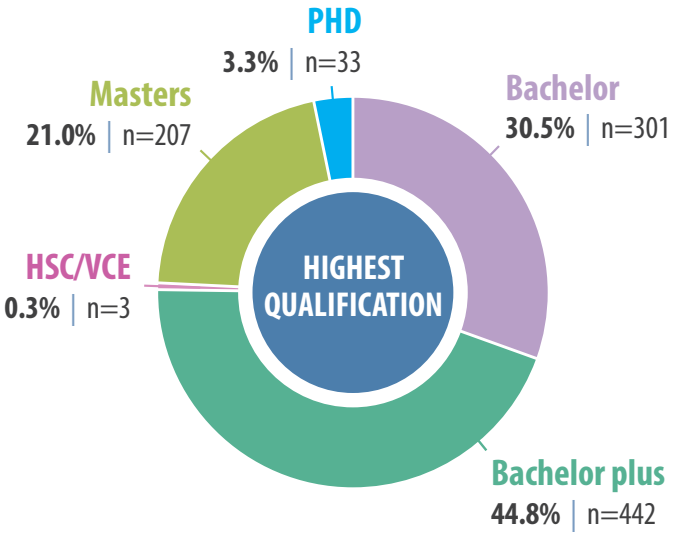
Study participants: well educated, but not necessarily in mathematics, or members of professional associations for mathematics teachers

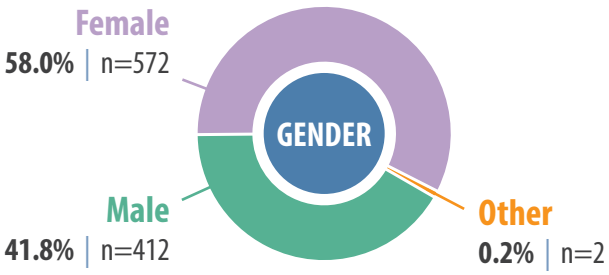
The figures below show the descriptive statistics for the sample in the teacher questionnaire. The sample was 58 per cent female, with an average of 15.6 years teaching and an average of 14 years teaching mathematics. These distributions were right skewed, with a number of very long serving teachers (maximum 55 years). The median number of years teaching was 13, with a median of 10 years teaching mathematics.

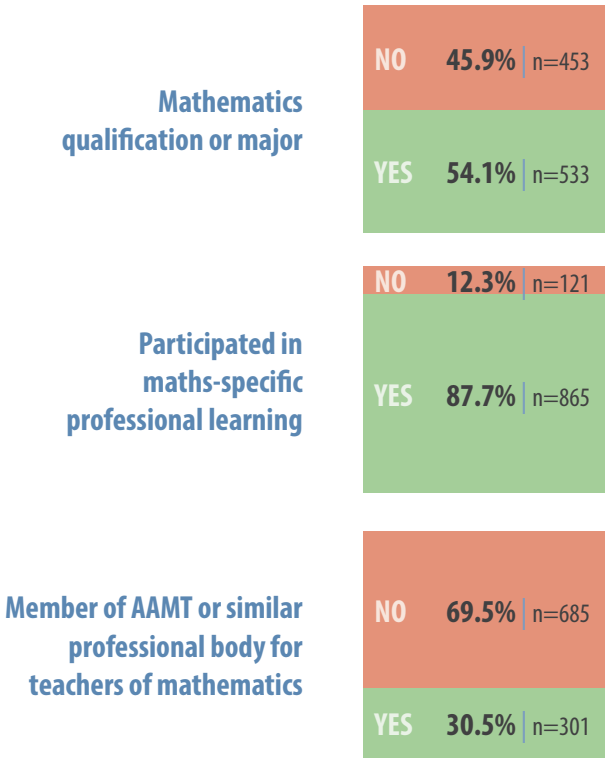
Over 99 per cent of research participants had at least a Bachelor degree, with nearly 70 per cent holding more than a Bachelor degree: e.g. Bachelor plus a Graduate Certificate in applied field, Masters, or a PhD or Doctorate.

Only 54 per cent of participants majored in mathematics or held a mathematics qualification, indicating that a considerable proportion would be considered an "out of field" teacher, consistent with the findings of O'Connor and Thomas (2019) and TIMSS data (2015) which found 22 per cent of Australian Year 8 students were taught by out-of-field teachers, much higher than the international average of 13 per cent.

Most teachers in the sample (88 per cent) had completed some kind of maths-specific professional learning or development programs, but only 30.5 per cent were a member of a professional body.







	N	MISSING	MEAN	SD	MIN	Q1	MEDIAN	Q3	MAX
Years Teaching	972	14	15.63	11.76	1	6	13	25	55
Years Teaching Maths	965	21	13.97	11.39	1	5	10	21	55

Overall, mathematics teachers in this study were confident

Overall the mathematics teachers in the study were very positive, with most nominating that they mostly agree/strongly agree with the statements below:

- I can get my students to follow the classroom or school rules
- I am able to effectively use a variety of assessment and feedback strategies
- I am able to access a wide variety of classroom resources that support my teaching content and pedagogy
- I am confident when it comes to demonstrating, modeling and explaining mathematical skills and concepts to my students

Most participants also indicated positive self-efficacy around their teaching, and agreement on the importance of deep learning. Participants were slightly less positive on the questionnaire items about "teachers in my school" compared to themselves, with more opting for "slightly agree" or "mostly agree" rather than "strongly agree" in their answers to questionnaire items. This sample could be biased due to the opt-in nature of the questionnaire, with more confident teachers being more inclined to participate. Table 1 in the Appendix B shows the responses to the teacher questionnaire items about self-efficacy, the teaching environment, and views on teaching.

Teacher confidence bolstered by relevant qualifications and professional learning

Teachers who had participated in maths-specific professional learning or development were nearly twice as likely to strongly agree they could ***access a wide variety of classroom resources that support [their] teaching content and pedagogy***.

The teachers that had participated in maths-specific PL were also more confident that they were "able to effectively use a variety of assessment and feedback strategies", "confident when it comes to demonstrating, modeling and explaining mathematical skills and concepts to my students", and "get my students to follow the classroom or school rules".

Teaching confidence was also positively related to having a qualification in mathematics with 64 per cent of maths-qualified teachers strongly agreeing that "I am confident when it comes to demonstrating, modeling and explaining mathematical skills and concepts to my students" compared to 39.7 per cent of teachers who would be considered to be teaching "out of field". Those teachers who had a mathematics qualification were also more likely to agree that "It is important to teach (and for learners to know) the material that will be covered in exams".

Indicators such as years of teaching, gender, and highest qualification did not have a significant relationship with the self-efficacy measures in the teacher questionnaire. This analysis used chi-squared tests of independence to measure the statistical significance of the relationships between items in the teacher questionnaire.

Student perception survey results reveal student-teacher connection as a mega-factor of effective teaching

A validation of the Pivot results for the mathematics teachers in the survey shows that a student-teacher connection appears to be a higher-order factor for student learning and engagement, and also teacher engagement. It is speculated that students' perceptions of their teachers' effectiveness are heavily influenced and filtered by the quality of student-teacher interactions, a phenomenon that has been identified in other research ((Hamre et al 2014; Kufheld 2017; Wallace et al 2016)). A critical implication when considering students' experience in the classroom and of their teachers is that when the quality of those interactions are weaker, this is likely to affect students' perceptions of most if not all of their teachers' individual practices.

Table 2 in the Appendix B shows the Pivot scores for the 673 survey participants who had received student feedback from maths classes in 2019. The average student feedback scores for mathematics teachers were generally similar to the population norms, with the largest differences seen in the items "This class keeps my attention - I don't get bored", "This teacher makes what we are learning interesting", and "In this class, the teacher helps me build my vocabulary" - each of which had a lower average with a notable, albeit small, effect size (Cohen's d of 0.2 to 0.25), than the population. The latter result could possibly being a product of the content taught in secondary school maths curriculums, while the former may be related to the views of an average student on their maths classes relative to other classes. (Compulsory subjects tend to receive lower scores than elective subjects, reflecting perhaps research that greater student voice and choice in what and how they learn has positive impact on engagement and learning outcomes (Quaglia 2016; Hattie 2009). The rank of the questions were also generally similar between the mathematics teacher sample and the population norms.

Follow-up qualitative interviews shed light on the connection between robust teacher-student connections and maintaining student engagement and learning growth, as well as some of the risks when teaching mathematics:

> *"Maths as a subject is one where you can fall into a trap of being very procedurally-focused; in which students gain the view that it's very black and white and has no room for creativity. And some teachers are able to teach effectively this way. However, maths teachers need a very deep conceptual understanding of the concepts, so that the student's procedural understanding can be enriched by the conceptual.* ***A deep conceptual understanding also enables maths teachers to be flexible and welcoming of student-led discussions or suggestions of alternative procedures.*** *"That's interesting, let's see why that worked", instead of "no that's not the way I showed you".*"
> Participant #1512337

> ***[Maths teachers] need a very good understanding of the maths themselves, and ability to articulate it clearly for each student, and understanding of a myriad of different ways that kids can learn it, and how to hook it in to kids thinking.*** *This is because so many kids find it difficult, and it's vital to develop their confidence and engagement. This all comes from a good relationship, knowing your kids, and them knowing you know them, care for them and will support them.*"
> Participant #1605142

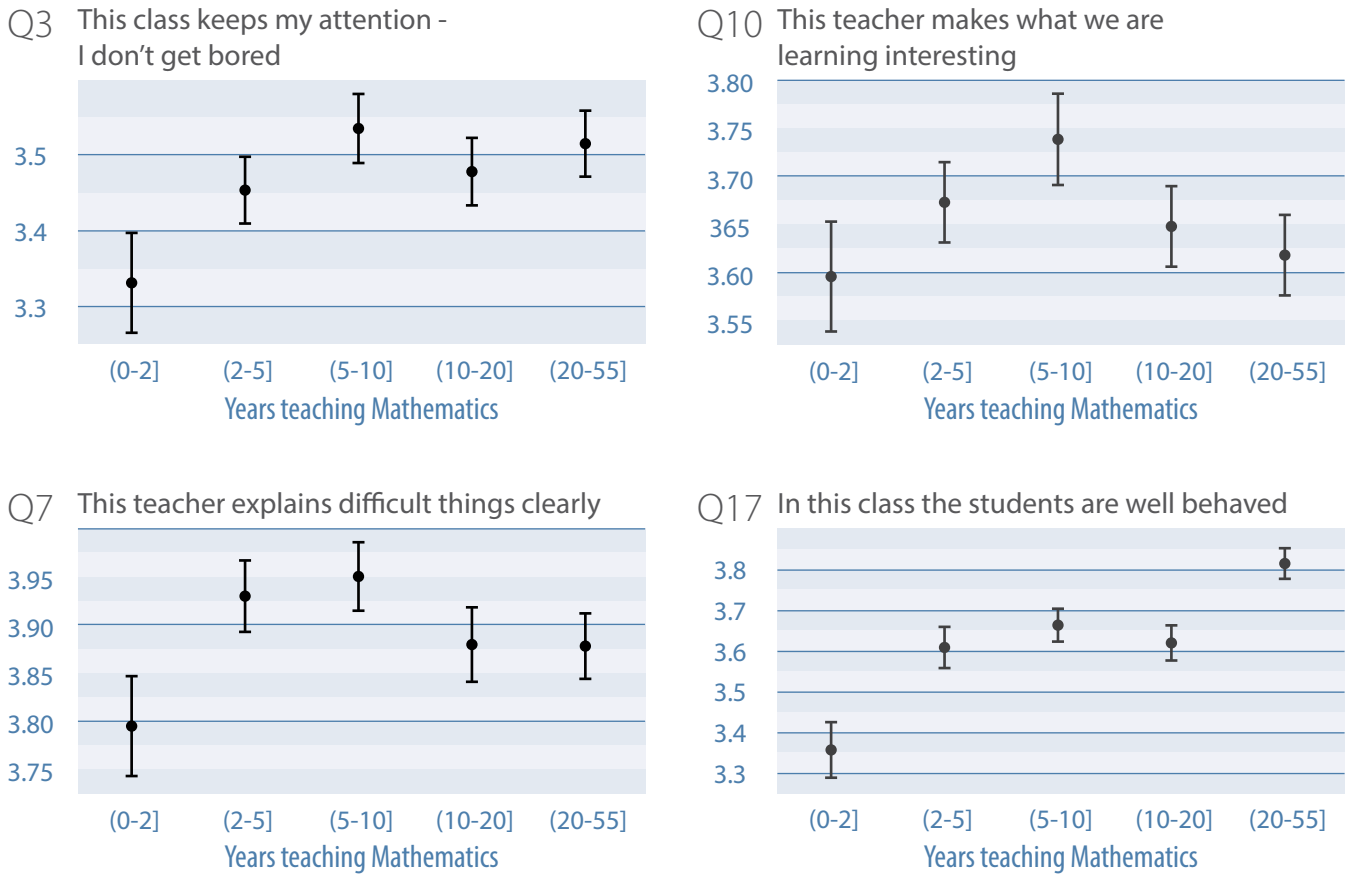
Asked what strong teacher-student connection look like, the Top Teachers elaborated:

> *"Very open, honest dialogue, supportive ...A really positive relationship is key to all of this, so kids believe that you are there for them to get them where they want and need to be. Moving around and questioning them.* ***A super-interactive environment.***"
> Participant #1605142

Less-experienced teachers tend to encouter more challenges with classroom management and engagement

Figure 2 shows the student perceptions that varied most by the number of years of mathematics teaching experience. The graphs show the mean scores from the items, with the error bars showing the standard error for that group. **Teachers who had taught mathematics for less than two years scored lower on student survey items relating to keeping a student's attention, explaining difficult things clearly, student behaviour, and making learning interesting**. A follow up one-way ANOVA showed that the mean difference between teachers with under two years of experience and the other groups was statistically significant for keep student attention, explaining difficult things, and students' in-class behaviour. Students rated teachers with between five and 10 years most highly on the first three, and the most experienced teachers had students reporting the best-behaved classrooms.

Figure 2
Student perceptions of effective teaching, by years of teacher experience teaching mathematics



It is noteworthy that one of the items listed here relates specifically to student and classroom behaviour. Managing classroom behaviour of students is one of the recurrent themes highlighted in multiple major studies of graduating and early career teachers. The SETE Project found that graduate teachers felt underprepared in classroom management compared to nearly all other elements of their teaching (Mayer et al. 2015). The most recent OECD report revealed that just 45 per cent of the thousands of Australian teachers surveyed reported that they felt prepared to manage their classrooms (2019). The data suggests a need for further improvement and emphasis in pre-service teacher education as well as early career teacher support, including teachers of maths, in more effective classroom management strategies. This hypothesis was confirmed in follow-up interviews with teachers.

A Top Teacher with 43 years experience emphasised that mathematics teachers need to be "100 per cent confident" in managing behaviour if they want to support kids to learn, saying:

> *"If you can't manage behaviour, if the class isn't a positive learning environment, the kids won't be able to engage and learn and make progress."*
> Participant 1605143

Another Top Teacher with 20 years experience teaching, plus experience as a head of a mathematics department in a school noted that she wasn't ready for the classroom management challenges she encountered in schools.

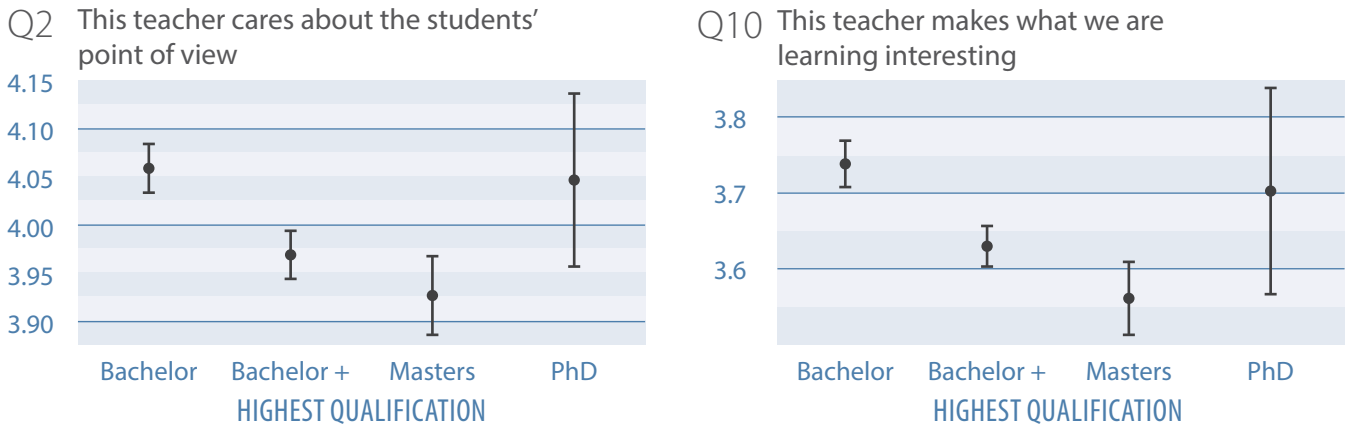
> *"When you're an early career teacher, you're pretty much thrown in the deep end very quickly by yourself in mainstream schools. I'd like to see greater investment in new teachers having opportunity to team teach for a while, not be the only teacher in the classroom, and not just for the first five weeks with mentor."*
> Participant #1530830

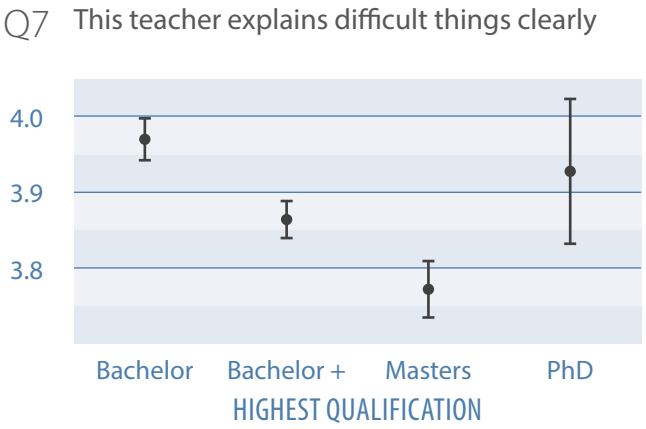
These findings are broadly consistent with other Australian studies on teaching, which indicate that teaching quality does seem to improve with experience but only for the first five or so years in the classroom (e.g. Rice 2003), and that after that point, experience accounted for less than one per cent of the variation in teaching quality (Leigh 2010). It also aligns with international research suggesting that the effects of experience are at best small and at worst statistically insignificant (Goldhaber and Anthony 2007; Goldhaber and Brewer 2000). It reinforces that more can and should be done to support teachers in their first five years, especially when it comes to creating and sustaining productive learning environments in their classrooms, a prerequisite to deeply engaging with mathematics.

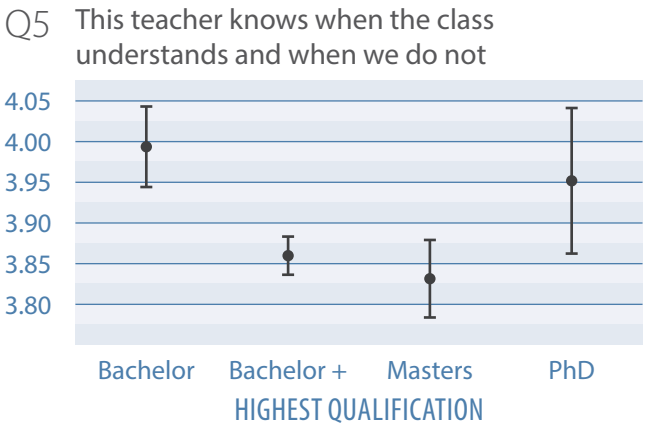
Higher qualifications of teachers can have an inverse (negative) correlation

Figure 3 shows the Pivot Student Perception Survey items that varied most by highest qualification. Relative to Bachelor-qualified teachers, those with Bachelor plus a Graduate Diploma and Masters-qualified teacher groups scored lower, on average, on the items relating to student understanding and interest. The pattern doesn't extend to the PhD group, but it should be noted that the PhD group is relatively small (which also accounts for the larger standard errors).

Figure 3
Student perceptions by teacher highest qualification







It has been found that experts can lose their sensitivity to how difficult a task can be for novice learners, which can increase the difficulty they experience in teaching skills and concepts to their students, a phenomenon known as "the curse of knowledge" (Hattie and Yates 2013). Given that the Pivot survey items are consistent with teacher sensitivity to learner needs and interests, it may be the case that some teachers with expert knowledge of maths find it challenging to fully understand and respond to the potential misconceptions and challenges their novice learners grapple with, or were still in their early years of teaching.

Two of the Top Teachers in this study were primary school teachers who later moved into secondary school mathematics teaching roles. These teachers undertook university-provided, government-funded graduate diplomas in mathematics teaching, which they rated very highly and emphasised were extremely valuable equipping them with the relevant conceptual knowledge as well as strategies and tools to communicate these concepts to students of different ages, abilities and levels of engagement. These teachers also stressed that the fact that the course went for a year or two, involved face-to-face time, and opportunities to deeply reflect, apply, refine and collaborate with peers in the course, contributed to this training being so very useful.

> *"[the Graduate Diploma in teaching maths to students in years 7-10] provided me with great data and resources and strategies.* ***Most definitely the best PL for maths I've done.*** *Weekend intensives. Made connections with other teachers from all over Victoria."*
> Participant #1512446

Research on the impact of teacher certification or registration on effective teaching has been inconclusive, with some studies finding strong positive effects (Darling-Hammond 2000), and others small, mixed or negative effects (Rice 2003; Wayne and Youngs 2003). Wayne and Youngs' review noted that mathematics teachers, unlike those in other subjects where results are unclear, do produce better results for their students across these types of measures of subject area knowledge and experience, and our results suggest that specialised training in mathematics instruction appears to be the key factor (as opposed to teaching more generally, or in other fields).

This study did not find significant relationships between a teacher's gender, participation in professional learning programs, or possessing a mathematics qualification, and their student survey scores on different Pivot items. This gender finding is strikingly at odds with findings of studies on student perception surveys completed at universities of their lecturers and tutors which indicate discrimination against women (Mitchell and Martin, 2018; Fan et al. 2019). It is also borne out by analysis of the Pivot data set more broadly (including teachers of other subjects) which does not indicate that school students score teachers differently based on their gender.

The results on professional learning could be explained by the fact that not all professional learning programs are relevant to the needs of those teachers or of their students, or that the PL is not reinforced with time for teachers to apply and refine the learning and strategies taught, or time to discuss with colleagues. It is hypothesised that where these features are present, the PL is highly valuable and supports quality teaching, but where PL is a "one-off" session with minimal application, minimal collaboration and practice opportunities, and no follow-up, it had negligible or negative influence on teaching and learning. These hypotheses were confirmed in qualitative interviews:

> *"I don't really value "sit and listen" PL. **The most valuable PLs are things you can easily link in to the classroom and put into practice**, such as lesson observation, discussing feedback with a colleague, then implementing in class."*
> Participant #1512337

> *"**The best PL I ever did had time after the PL to follow-up**, to explore how to embed, to check-in with course teachers."*
> Participant #1530830

> *"[Some teachers need] more support in conveying maths concepts to students. And others also need conceptual training in maths. **PL needs to be tailored** in the same way that teachers tailor lessons and activities for their students, scaffolding them."*
> Participant #1512337

Teacher versus student perceptions

The final component of quantitative analysis was an examination of the relationships between teacher perceptions from the teacher questionnaire and student perceptions as measured by the Pivot instrument, with particular focus on items that measured similar concepts.

The pattern that emerged from the data was there seem to be two distinct cohorts of teachers who tend to receive higher student perception ratings - the teachers who have the highest self-efficacy scores (rating themselves with 'strongly agree') and a smaller group who were rating themselves with 'strongly disagree' on the efficacy questions. This could suggest that some mathematics teachers suffer from "Imposter Syndrome" where they don't believe they are doing a good job (or at least are not confident) despite the higher ratings from their students. It could also suggest that some students are overstating effectiveness. In other words, while student perception is a reliable indicator of effective practice, at an individual or small sale it is not infallible), which is why Pivot recommends whole class provides feedback, rather than a hand-picked handful of students. This reinforces the need for clear, strong data drawn from multiple measurement instruments - including Pivot's Student Perception Survey, for a fulsome understanding of effective teaching.

Figure 4 plots the mean scores for *This teacher is knowledgeable about the topics in this subject*, with standard error bars, against the survey item *I am confident when it comes to demonstrating, modeling and explaining mathematical skills and concepts to my students* (1=Strongly Disagree, 6=Strongly Agree). The highest performing groups are the teachers who answered in the extreme, while the worst performing group are the "slightly disagree" cohort (corresponding with 3 in Figures 4 and 5).

Figure 4 Teacher confidence vs Student Perception of Teacher Knowledge

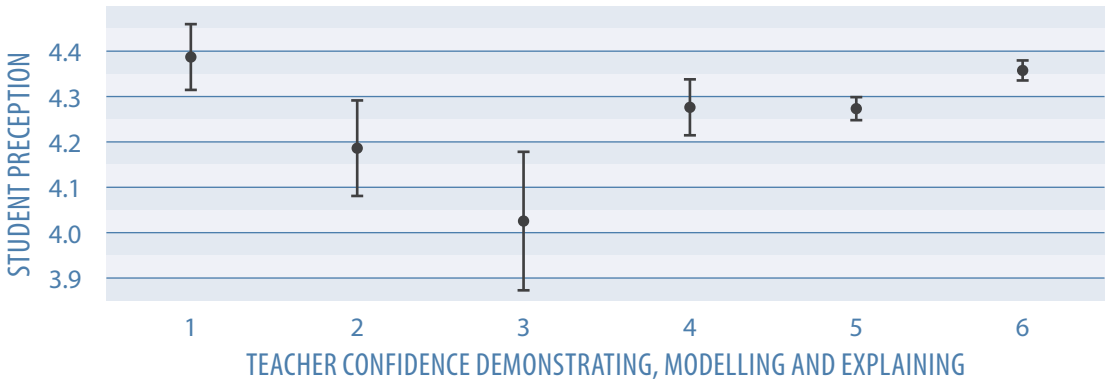
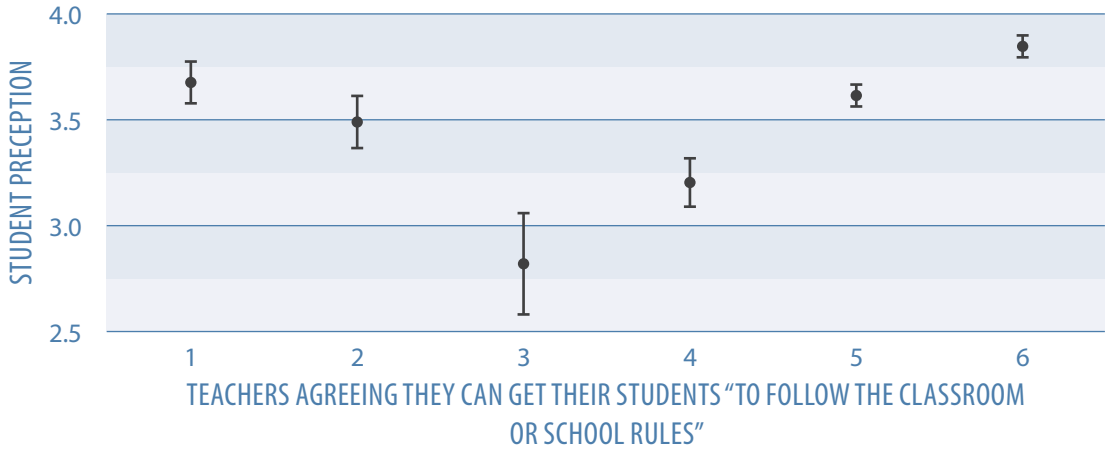


Figure 5 plots the classroom control items, *In this class, the students are well behaved* (1=Strongly Disagree, 6=Strongly Agree), from the Pivot student results, against the teacher survey item, *I can get my students to follow the classroom or school rules*. Again, the "slightly disagree" cohort has the lowest average scores whilst the teachers at each extreme have the highest.

Figure 5 Student and Teacher Perceptions of Student Behaviour



Pivot student survey results are discussed further in implications, but the project was able to confirm the factor structure of the five professional standards Pivot groups its student perception survey questions within with high estimates of reliability (Standard One alpha = .95, Standard Two alpha = .94, Standard Three alpha = .95, Standard Four alpha = .90, Standard Five alpha = .95.).

The noteworthy findings here are that students view classroom management as more distinct than any other element of their teachers' practices, and that strong, positive teacher-student connections are a mega-factor of effective teaching and learning.

Teaching beliefs

Items on self-efficacy and collective efficacy, as well as on learning, were gathered and explored in our study, to probe findings of earlier studies that found that teachers' beliefs significantly influence their planning instructional decisions and classroom practices, including behaviour management (Kagan 1992, Pajares 1992, Calderhead 1996, Hattie 2003).

The items in the teacher questionnaire that had been hypothetically labelled as self-efficacy and collective teacher efficacy grouped together with no obvious sub-factors. These items are in Appendix A1. Estimates of reliability were sufficiently high to have confidence in the total scores from the scales (self-efficacy alpha = .88, collective teacher efficacy alpha = .87). This is consistent with earlier research finding more effective teachers tend to have *higher self-efficacy* (Woolfolk, Rosoff and Hoy 1990; and Henson 2001, Ross 1994, Ross 1998, Ashton and Webb, 1986, Muijs and Reynolds 2001).

Beliefs about teaching and learning were originally segmented based on deep versus surface, with teachers asked how much they agreed with the four statements below:

- Learning is building up knowledge by acquiring facts, information and skills
- It is important to teach (and for learners to know) the material that will be covered in exams.
- Learning is about seeing things in a more meaningful way.
- My teaching helps students to apply information to new situations or problems

Factor analysis grouped all items together into just one factor that united surface and deep beliefs about teaching and learning, with high estimates of reliability (alpha = .83). This finding indicates that in **general, teachers who strongly endorse surface views of teaching and learning are also much more likely to endorse deep views, as opposed to viewing these as mutually exclusive**. Between 85 per cent and 90 per cent of participants agreed that both deep and surface components of teaching and learning were important. This result aligns with earlier studies finding an empirical link between improved student outcomes teachers that follow both deep and surface focused teaching and learning beliefs (Entwistle 1997, Biggs, Kember et al. 2001, Biggs and Collis 2014).

This finding is novel, because it shows that teachers who push for deep versus surface teaching and learning are not noticeably more effective by any measure (self-efficacy, Pivot results, etc.). Further, it is noteworthy that mathematics teachers do not see surface learning and deep learning as dichotomous, but instead consider surface knowledge (facts) and deep learning (concepts) as both integral to good teaching. Again, our qualitative interviews shed light on this phenomenon, with a disjuncture between "best practice" teaching and "exam and curriculum driven teaching". **Top Teachers described excellent mathematics teaching as combining explicit instruction with guided inquiry, individualised tasks, choice, and collaborative exercises to explore and master concepts and skills. But they all emphasised this was very difficult to do in Year 11 and 12 classes**, where the extensive content that needed to be covered allowed little time to delve deeply into the concepts or their application. When it came to Years 11 and 12, there was a perception that "good teachers" were those that managed to get through the content required for the exams:

> *"I still get really frustrated that we have different system **Years 7-10 (where there is lots of individualised student learning and more time to explore and practice), then the regimented approach for Years 11 and 12**. The policy for mathematics needs to change. I'm putting in so much extra time, ploughing through everything required for their exams [but it's] sink or swim as there's so much to get through. You do your best to support the kids individually, but too much curriculum to cover to do effectively."*
> Participant #1605142

> ***"Good teaching and learning involves the opportunity to wonder, and to practice techniques that arise from this wonder, and balancing conceptual discovery and explicit teaching of theory, connections and practice…[but] there is a gap in between what makes good teaching and the systemic requirements of Years 11 and 12, which are in conflict."***
> Participant #1530830

> [what does good mathematics teaching and learning look like?] *"**Explicit modelling, but with a strong element of inquiry as well [and] having students involved**. Prioritise them thinking about what is being taught, rather than giving simple answers. Ask them questions starting with "what if" and "why", such as, 'What would it look like if? or 'Why did that work? [...Teaching senior years] is less scaffolded, and more about getting through content. I would love to spend more time with the students, deeply exploring the curriculum content, really breaking it down and exploring it with them, but then that would mean having to condense the next two lessons."*
> Participant #1512337

Policy implications: building and spreading expertise

Australia has many excellent teachers of mathematics, but our schools and systems and should do better at sharing excellent teaching practices of its expert teachers within schools, between schools, and across systems.

Results from this study indicate that while mathematics teaching expertise tends to grow with experience for the first decade or so, and that those with decades of experience were strongest on classroom management, some of the best teachers are new to teaching. This suggests that more and less experienced teachers can learn from each other. All Top Teachers emphasised their desire to more closely collaborate with colleagues and in particular to share their knowledge and skills. It was clear that they were not encouraged or provided with as many opportunities as they felt appropriate.

When asked if they had any recommendations, all the Top Teachers interviewed independently suggested more time and support for collaboration with other teachers (including after participating in professional learning), and more time to prepare lessons that best met their students' learning needs.

> *"Teachers are really time poor. More time to prepare for classes, so teachers don't need to take short-cuts. And this gives more time for teachers to sit down together to discuss and try new strategies and resources. The system should* ***invest properly in teacher collaboration and preparation****, because if you're slogging it out by yourself you're in real trouble."*
> Participant #1530830

> ***"First step is time for collegial discussions****, setting aside a time to discuss each topic or concept, something like an hour per topic, and build up a bank of strategies for explaining and demonstrating each concept to students."*
> Participant #1512337

These findings and recommendations are especially important given that this study's results reinforce that skill in teaching mathematics is not the same as knowing mathematics, and that the best preparation for mathematics teaching is a university-delivered qualification that unites the "what" with the how". This is because while deep conceptual knowledge is important, it is also vital that pre-service teacher education and professional learning for in-service mathematics teachers focus on how to articulate mathematical concepts and strategies to students, especially those struggling with confidence, to build their engagement, understanding and skills in maths. Our study reinforces the academic construct of 'Pedagogical content knowledge (PCK)' . Here we defer to the elegant explanations of two leading academics who have championed this approach.

> ***"If teachers are to be successful they would have to confront both issues (of content and pedagogy) simultaneously, by embodying the aspects of content most germane to its teachability****… It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction."*
> (Shulman, 1986, p. 8-9)

> *"It is an idea rooted in the belief that teaching requires considerably more than delivering subject content knowledge to students, and that student learning is considerably more than absorbing information for later accurate regurgitation. PCK is the knowledge that teachers develop over time, and through experience, about how to teach particular content in particular ways in order to lead to enhanced student understanding."*
> (Loughran et al. 2012)

In our study, PCK included being able to build creativity and collaboration into mathematics lessons, as a means to practice the application of mathematical skills and knowledge, respond to student suggestions, including alternative ways of solving tasks. It was also described as a way of deepening student engagement in mathematics and cultivating these critical capabilities as they apply in maths and beyond.

Lack of deep conceptual knowledge - more commonly found in out-of-field teachers - is more likely to result in a procedural approach to teaching mathematics. While this approach is certainly valid and useful, is best complemented by other strategies that can do more to foster and maintain engagement, such as mathematical challenges that prompt students to think creatively to solve "real world" problems either solo or in groups, be it the trajectories of goal-scoring kicks that feed into elite sports coaching, or estimating water levels for environmental actions.

The findings also indicated the vital importance of strong connections with each student, mutual respect and continuous interactions as a critical component in quality teaching, as this enables teachers to understand where each student is at, and how to support them. As one teacher expressed in the research interview, this is especially important for mathematics teachers, who:

> *"need a very good understanding of the mathematics themselves, and ability to articulate it clearly for each student, and understanding of a myriad of different ways that kids can learn it, and hook it in to kids thinking. This is because so many kids find it difficult, and vital to develop their confidence and engagement.* ***This all comes from a good relationship, knowing your kids,*** *and them knowing you know them, care for them and will support them."*

An ongoing concern in mathematics teaching is the growing need for high quality mathematics teachers as the number of school students skyrocket, students' maths knowledge and skills appear to be declining in both relative and absolute terms, and a sever shortage in qualified mathematics teachers. While this project does not offer solutions to address future workforce needs, it does suggest that a fuller understanding of what it means to be a quality mathematics teacher is long overdue. Although we may perceive teachers to be quite different on the basis of their experience, their highest degree, and whether or not they are traditionally trained to teach mathematics, students don't tend to view these teachers as particularly different from one another. Granted this is just one data point, but it is nonetheless an important one to consider. Yet, we know from the broader research that maths qualified teachers and those who participate in professional learning tend to produce better results for their students. There is a need to put this finding to the test in the Australian context to ensure that the training and qualifications are in fact aligning with better student outcomes, whilst simultaneously creating a space for mathematics teachers to consider the role that student voice and students' perceptions of teaching should have in shaping views on what constitutes excellent mathematics teachers.

Two examples of PCK and student-teacher connections in action in maths

Teachers may incorporate topics of students' interest and relevant challenges in their design of mathematics problems (some even incorporate the names of students.) Mathematics teachers may also leverage grouping and paired work strategically as a way to build or reinforce connections and collegiality between students or to leverage peer to peer support as an instructional intervention for both lower and higher achieving students. These teachers also utilise feedback and praise, specific to the learning in their discipline, regularly and effectively to ensure that students feel they have the support, encouragement and recognition they need from their teacher to be successful.

Effective maths teachers utilise an array of approaches to incorporate surface learning and deep learning to support students' development of knowledge and skills. One common approach is to utilise scaffolded questioning during instructions, using Bloom's Taxonomy verbs to elicit understanding and build conceptual knowledge from lower level to higher level questions. Another is to structure lessons to include a combination of routine activities such as a review of previously learnt material, including comparable problems (spiralling their curriculum), explicit instruction of a new skill with independent practice, and a mathematical inquiry or investigative activity that promotes more complex, critical, and creative thinking.

Priorities for professional learning

Student survey results highlight a few key implications for supporting teachers through PL. The first is the criticality of classroom and behaviour management. Well-managed classrooms are a prerequisite to effective instruction, and in the eyes of students' are the most distinct teacher practice. Less-experienced teachers tend to encounter more challenges in this area, so classroom management support tailored to the context of mathematics classrooms for the first few years of teaching will likely improve numerous areas of teaching and learning.

Additionally, students' perceptions of teachers can be quite different from teachers' perceptions of themselves as well as the perceptions of other adults. Supporting teachers to deepen their sensitivity to students is a major lever for improving instruction. This could involve focusing on areas such as eliciting students' thinking and responding to misconceptions, encouraging students to seek help and communicate when they do not understand, and strengthening the quality of teacher-student interactions in the classroom.

The most important factor for whether professional learning achieves desired outcomes is whether teachers participating saw it as relevant to them and their students. This was most apparent in the research interviews and accords with other research. Professional learning should more clearly articulate the benefits and relevance to mathematics teaching, particularly where it concerns student-teacher connections, creativity and collaboration in maths classrooms.

> *"The most important thing for professional learning is that teachers see it as relevant to them. There is a tendency for maths teachers to dismiss it as something that won't work in maths classroom, so demonstrating relevance to teachers is critical."*
> Participant #1605142

The best types of professional learning identified by teachers were ongoing and promoted a continuous cycle of action and reflection, both as an individual teacher, and with colleagues, including in the immediate period following the delivery of the PL, to support embedding or refinement of the practices in their classrooms.

Conclusion

This study reinforced that Australia has many excellent teachers of mathematics, but can do better at sharing excellent teaching practices of its expert teachers within schools, between schools, and across systems. The study also reinforced that mathematics expertise and teaching expertise are different but are both required for excellence in maths teaching as these two sets of expertise work in conjunction with each other. This has implications for both pre-service teacher education and ongoing professional learning.

Top Teachers of mathematics combined deep knowledge of key concepts with excellence in communicating these in engaging ways to students of different levels of understanding and interest. This excellence appears to be supported by a bedrock of strong student-teacher connections, which this study identified as a "mega-factor" for excellent teaching and learning. These connections are characterised by mutual respect and continuous checking-in with students to appreciate how well they understand the material and best next steps to sustain engagement and progress their learning.

There was a small but significant relationship between students' perceptions of their teachers' knowledge and teachers' self-reported levels of self-efficacy. While this only accounted for a small degree of the variance in Pivot results, it suggests that higher levels of self-efficacy positively contribute to better mathematical practice and student outcomes, a finding consistent with the large body of research on teacher self-efficacy to date.

Quantitative analysis indicated the impact of higher qualifications and professional learning showed up most strongly on teacher confidence, but with less effect on student perceptions, which aligns with national and international research on these indicators. This appears to be explained by the variations in quality of teaching degrees and maths-heavy degrees, and uneven quality and relevance of professional learning for mathematics teachers. Follow-up interviews emphasised the best preparation for teaching mathematics was maths-specific training as part of a teaching course run by a university/teaching college, either in their pre-service teacher education, or in a later qualification such as a Graduate Diploma in teaching mathematics, as this united the "what" with the "how" needed for pedagogical content knowledge. For professional learning to be useful, it had to be ongoing, directly linked to classroom practices, and include opportunities to reflect and discuss with colleagues. This allowed the PL to be implemented, and if needed, refined, in a way the best met the learning needs of students.

As stakeholders in education, teachers, schools, professional associations, school systems, universities and governments all have a role to play in building and sharing mathematics teaching expertise. These are listed in the Executive Summary, on pages four and five.

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Appendices:

Available upon request by emailing teachingmaths@pivotpl.com

Apendix A. Research instruments:

A1. Questionnaire completed by mathematics teachers in this study
A2. Pivot student perception survey on effective teaching practices
A3. Semi-structured interview questions asked of Top Teachers

Appendix B. Statistical compendium:

B1. Teacher questionnaire items and results
B2. Pivot Student Perception Survey on Effective Teaching Practices - Scores
B3. Teacher Beliefs Factors
B4. Descriptive Statistics on teaching beliefs
B5. Relationships in the maths teacher survey
B6. Relationships between student perceptions and teacher profile descriptors